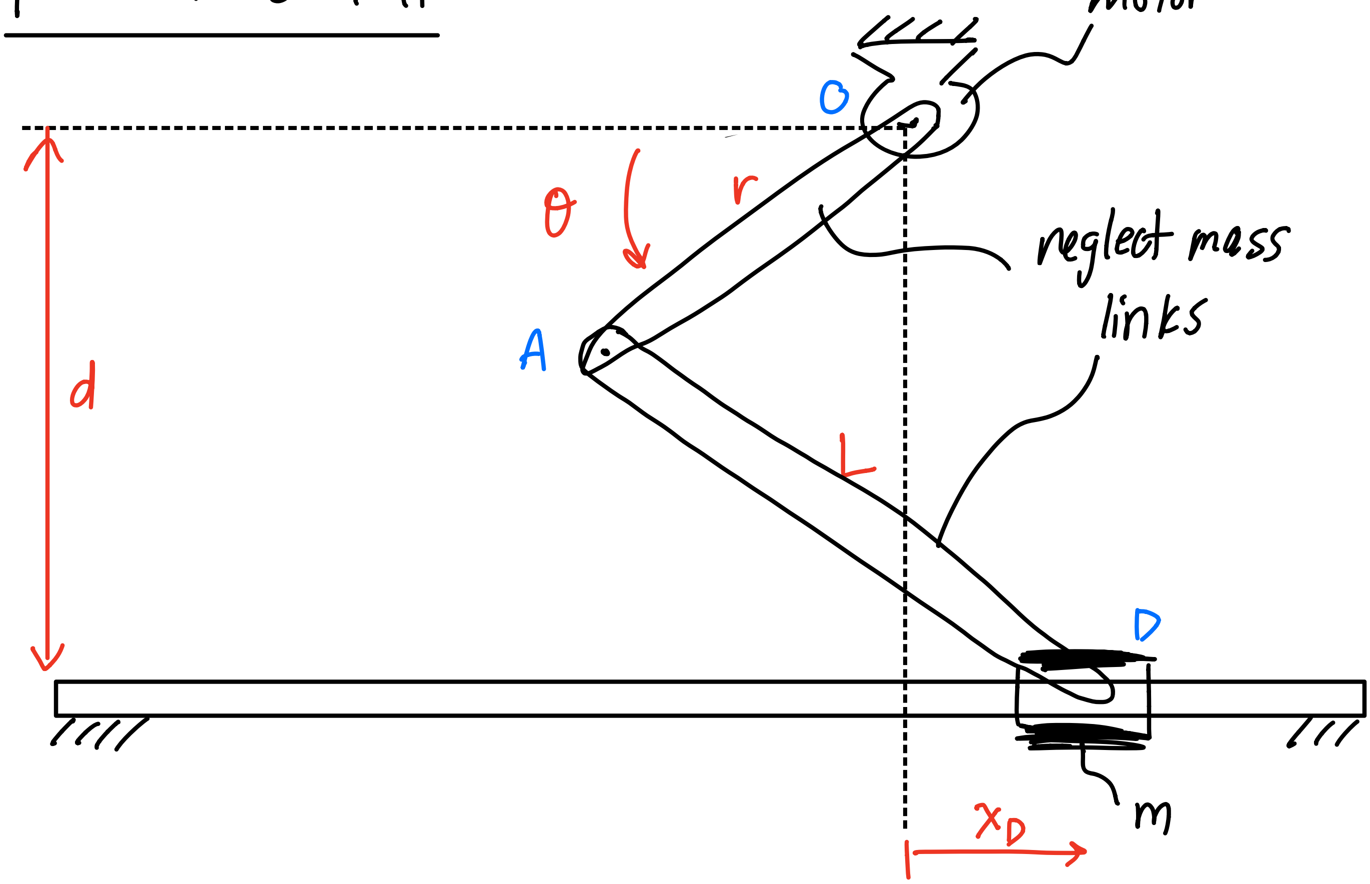
MAE 2030 Lecture #35:

5/5/21

Today: Mechanism example

Problem 18.4.11 (?):

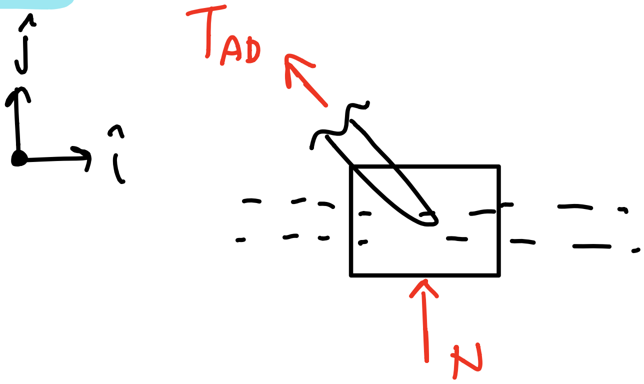
neglect g



Given: $\underbrace{\theta, \dot{\theta}, \ddot{\theta}}_{\theta(t)}$, r, L, d

$\boxed{T_{AD} = ?}$

FBD of mass:



LMB: $\sum \vec{F} = m\vec{a}$

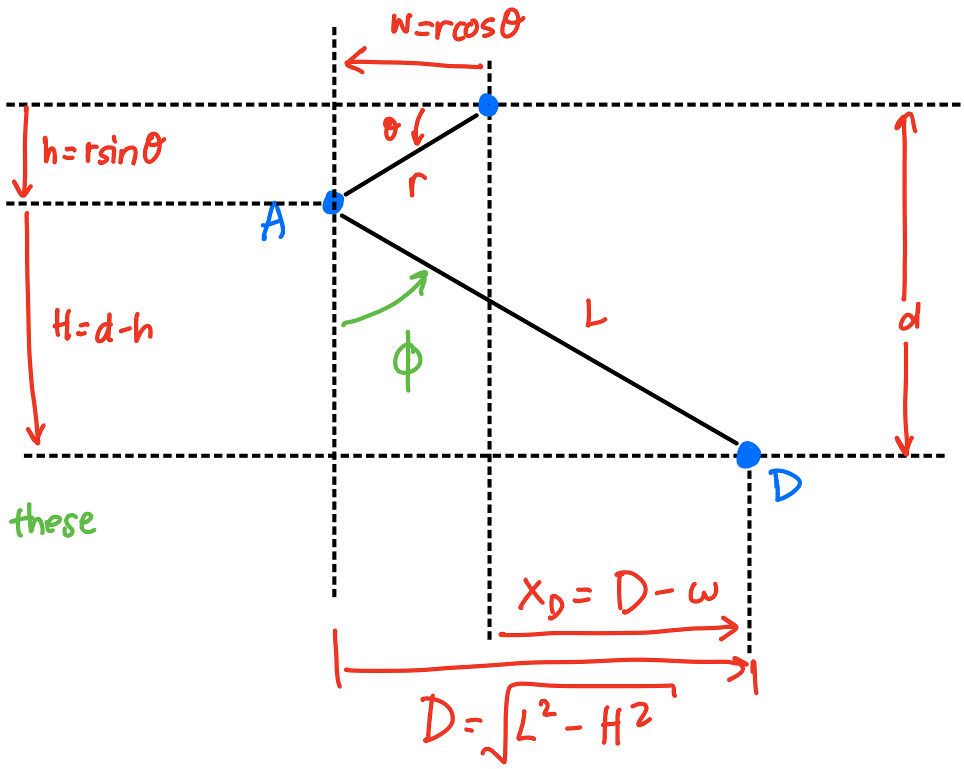
$$= m\ddot{x}_D\,\hat{i}$$

$$\left\{ N\hat{j} + -T_{AD}\,\hat{\lambda}_{AD} = m\ddot{x}_D\hat{i} \right\}$$

$\hat{\lambda}_{AD} = \vec{r}_{AD}/|\vec{r}_{AD}|$

$$\{\ \}\cdot\hat{i} \implies \boxed{-T_{AD}\,\hat{\lambda}_{AD}\cdot\hat{i} = m\ddot{x}_D} \quad (1)$$

Geometry & Kinematics to find: $\vec{r}_{AD}$, $\ddot{x}_A$



we know these

$w = r\cos\theta$

$h = r\sin\theta$

$H = d - h$

$D = \sqrt{L^2 - H^2}$

$x_D = D - w = $ mess $\rightarrow$ f(θ, parameters)

$$\vec{r}_{AD} = D\hat{i} - H\hat{j}$$

Method #1: $x_D = f(\theta, \text{parameters})$

$$\vec{v}_D = v_D \hat{i} \qquad v_D = \frac{df}{d\theta}\dot{\theta}$$

$$\vec{a}_D = a_D \hat{i} \qquad a_D = \frac{d^2 f}{d\theta^2}\dot{\theta}^2 + \frac{df}{d\theta}\ddot{\theta}$$

messy!

$$a_D \Rightarrow \ddot{x}_D \underset{\uparrow\ 1}{\Rightarrow} T_{AD} \checkmark$$

Method #2: use the instantaneous velocity & accel. relations

$$\vec{v}_D = \vec{v}_D$$

$$\vec{a}_D = \vec{a}_D$$

velocities first: $\vec{v}_D = \vec{v}_D$

$$\dot{x}_D \hat{i} = \underbrace{\vec{v}_A} + \underbrace{\vec{v}_{D/A}}$$

$$\left\{ \dot{x}_D \hat{i} = \underbrace{\dot{\theta}\hat{k} \times \vec{r}_{A/O}}_{\vec{r}_{A/O} = -r\cos\theta\,\hat{i} - r\sin\theta\,\hat{j}} + \underbrace{\dot{\phi}\hat{k} \times \vec{r}_{D/A}}_{\vec{r}_{D/A} = D\hat{i} - H\hat{j}} \right\} \quad (2)$$

$$\{\ \} \cdot \hat{j} \Rightarrow 0 = -r\dot{\theta}\cos\theta + D\dot{\phi}$$

$$\Rightarrow \boxed{\dot{\phi} = \frac{\dot{\theta} r \cos\theta}{D}}$$

~ plug $\dot{\phi}$ into ② ~

$$② \Rightarrow \dot{x}_D$$

<u>accelerations :</u> $\vec{a}_D = \vec{a}_D$

$$\ddot{x}_D \hat{i} = \underbrace{\vec{a}_A} + \underbrace{\vec{a}_{D/A}}$$

$$\{ \ddot{x}_D \hat{i} = \underbrace{\left(\ddot{\theta} \hat{k} \times \vec{r}_{A/O} - \dot{\theta}^2 \vec{r}_{A/O} \right)} + \underbrace{\left(\ddot{\phi} \hat{k} \times \vec{r}_{D/A} - \dot{\phi}^2 \vec{r}_{D/A} \right)} \} ③$$

$\{\} \cdot \hat{j} \Rightarrow \ddot{\phi}$ *can also take derivative of $\dot{\phi}$ b/c expression has symbols*

$$\{\} \cdot \hat{i} \Rightarrow \ddot{x}_D \xrightarrow{LMB} T_{AD}$$

demo: solving problem on MATLAB