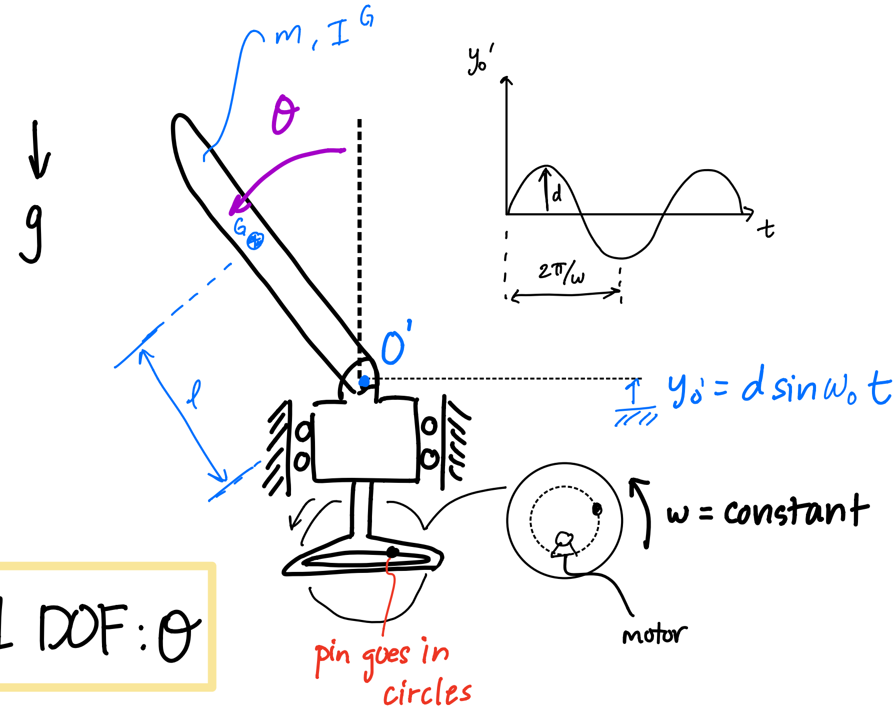
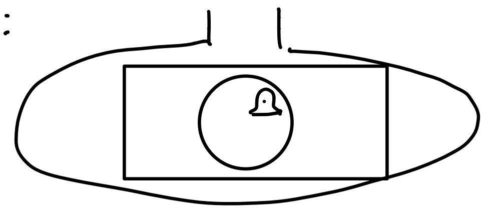
MAE 2030 Lecture #36: 5/7/21

Today: Inverted shaken pendulum



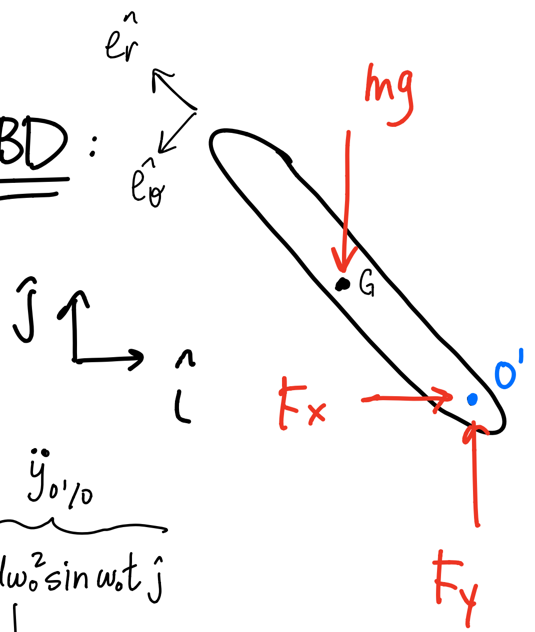
Mechanism: Scotch Yolk

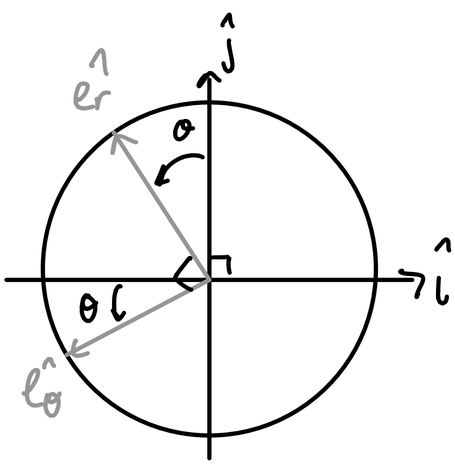
Alt:



$\boxed{1\ \text{DOF}: \theta}$

FBD:





$$\vec{a}_{G/O'} = -\omega^2 \underbrace{r_{G/O'}}_{\ell \hat{e}_r} + \dot{\omega}\hat{k} \times \underbrace{r_{G/O'}}_{\ell \hat{e}_r} \qquad \vec{a}_{O'/O} = \overbrace{-d\omega_0^2 \sin\omega_0 t\, \hat{j}}^{\ddot{y}_{O'/O}}$$

$$\underline{AMB_{/O'}}: \sum_i \vec{M}_{/O'} = \vec{r}_{G/O'} \times m\, \vec{a}_{G/O} + I^G \dot{\omega}\hat{k}$$

$$mg\ell\sin\theta\,\hat{k} = \ell\hat{e}_r \times m\left[\left(-\dot{\theta}^2 \ell\hat{e}_r + \ddot{\theta}\ell\hat{e}_\theta - d\omega_0^2 \sin(\omega_0 t)\hat{j}\right)\right] + I^G \ddot{\theta}\hat{k}$$

$$\left\{ mg\ell\sin\theta\,\hat{k} = \left[\vec{0} + \ddot{\theta}\ell^2\hat{k} + d\ell\omega_0^2 \sin(\omega_0 t)\sin\theta\,\hat{k}\right]m + I^G\ddot{\theta}\hat{k} \right\}$$

$$\{\ \}\cdot\hat{k} \Rightarrow mgl\sin\theta = l\underbrace{d\omega_0^2\sin(\omega_0 t)}_{-a_{0'}}\sin\theta\, m + \ddot{\theta}(ml^2 + I^G)$$

$$\vec{a}_{0'} = a_{0'}\hat{j}$$

$$-d\omega_0^2\sin(\omega_0 t)$$

$$\dot{\omega} \rightarrow \ddot{\theta} = \frac{m(g + a_{0'})\,l\sin\theta}{I^G + ml^2}$$

$$\dot{\theta} = \omega$$

ready for MATLAB

special case: $I^G = 0$ (point mass)

$$\ddot{\theta} = \frac{(g + a_{0'})\sin\theta}{l}$$

demo: Andy uses a jigsaw to demonstrate problem & explains why the stick stays vertical with enough shaking