MAE 2030 Lecture #37:

5/10/21

Today: Fundamentals Review ← classical mechanics

Assume (*)

* a point of mass is at a definite place $\vec{r}_i$ at time t

$$\vec{r}_i \equiv \vec{r}_{i/0}$$

* space follows rules of Euclidian/Cartesian geometry
* mass neither appears nor disappears
* consider "closed systems" only (fixed mass)
* Force $\vec{F}$ is the means of interaction between systems
* There is a reference system in which laws of mechanics are true
* Laws of mechanics apply to any closed system or subsystem
* The laws of logic & math apply

Define: (□)

all particles in system

□ $m = m_{tot} \equiv \sum m_i$

- [ ] $\vec{v}_i \equiv \mathring{\vec{r}}_i = \frac{d}{dt}\vec{r}_i \quad \Big\| \quad \vec{a}_i \equiv \mathring{\vec{v}}_i = \frac{d}{dt}\vec{v}_i$

- [ ] $G = \text{CoM} = \oplus : \; m_{tot}\,\vec{r}_G \equiv \sum m_i \vec{r}_i$

- [ ] $\vec{r}_{A/B} \equiv \vec{r}_A - \vec{r}_B \quad \Big\| \quad \vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$

$$\vec{a}_{A/B} = \vec{a}_A - \vec{a}_B$$

Linear momentum

- [ ] $\vec{L} \equiv \sum m_i \vec{v}_i$

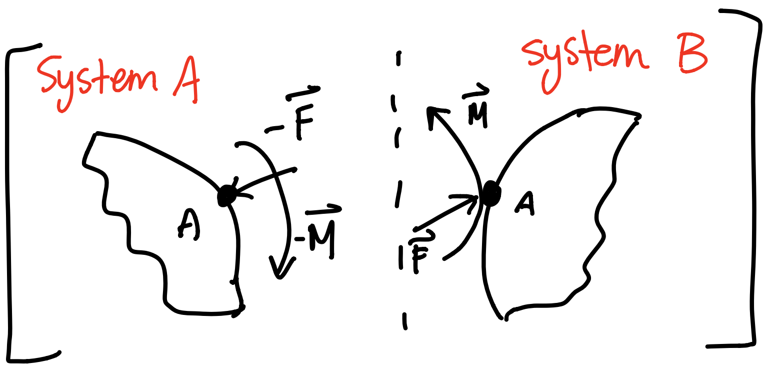
- [ ] $\vec{H}_{/C} = \sum_i \vec{r}_{i/C} \times m\,\vec{v}_{i/C}$

- [ ] $E_K = \sum_i \frac{1}{2} m_i \overbrace{\vec{v}_i \cdot \vec{v}_i}^{|\vec{v}_i|^2}$

Assume: (Basic Postulates)

* Action & reaction between systems

partial FBDs



* LMB: $\boxed{\sum_{ext} \vec{F} = \mathring{\vec{L}}}$

$* \text{AMB}: \quad \sum_{i \text{ ext forces}} \vec{r}_{i/C} \times \vec{F}_i = \boxed{\sum \vec{M}_{/C} = \dot{\vec{H}}_{/C}}$

if C is fixed in space anywhere $= \dot{\vec{r}}_{C/O} = 0$

any such C

Facts/Theorems: (☻)

- $\vec{L} = m\vec{v}_G$

- $\vec{H}_{/C} = \underbrace{\vec{r}_{G/C} \times m_{tot}\vec{v}_G}_{\vec{H}_{G/C}} + \underbrace{\sum \vec{r}_{i/G} \times m\vec{v}_{i/G}}_{\vec{H}_{/G}}$

- $\dot{\vec{H}}_{/C} = \underbrace{\vec{r}_{G/C} \times m_{tot}\vec{a}_G}_{\dot{\vec{H}}_{G/C}} + \underbrace{\sum \vec{r}_{i/G} \times m\vec{a}_{i/G}}_{\dot{\vec{H}}_{/G}}$

only true if C is fixed, moving at constant velocity, or $C = G$

useful

- $\sum \vec{M}_{/C} = \vec{r}_{G/C} \times m\vec{a}_G + \sum \vec{r}_{i/G} \times m_i \vec{a}_{i/G}$

  $\vec{a}_{G/O}$

  for any point C (can be moving)

- $$\underbrace{\vec{F}_i \cdot \vec{v}_i}_{\equiv \text{ Power}} = \frac{d}{dt}\frac{1}{2} m_i |\vec{v}_i|^2$$

- $$\underbrace{\int \vec{F} \cdot dr_i}_{\equiv \text{ Work}} = \Delta E_K$$

- For any system $\Longrightarrow$ 3 independent EoM (for 2D)

  ex) LMB + $AMB_{/C}$

  (2) (1)

  any vector not $\perp$ to line $\overline{CD}$

  ex) $AMB_{/C}$ + $AMB_{/D}$ + $LMB \cdot \hat{\lambda}$

  (1) (1) (1)

  ↳ $\hat{\lambda} \cdot \vec{r}_{CD} \neq 0$

ex) $AMB_{/C} + AMB_{/D} + AMB_{/E}$

$\quad (1) \qquad (1) \qquad (1)$

C, D, E not colinear

Any more than 3 eqns $\Longrightarrow$ redundant info

- $\vec{H}_{/G} = I^G \dot{\omega} \hat{k}$

rigid object

- ☐ $I^G = \int r^2 \, dm$

- ☐ $\vec{w} = \omega \hat{k}$ such that $\vec{v}_{C/D} = \vec{\omega} \times \vec{r}_{C/D}$ on a rigid object