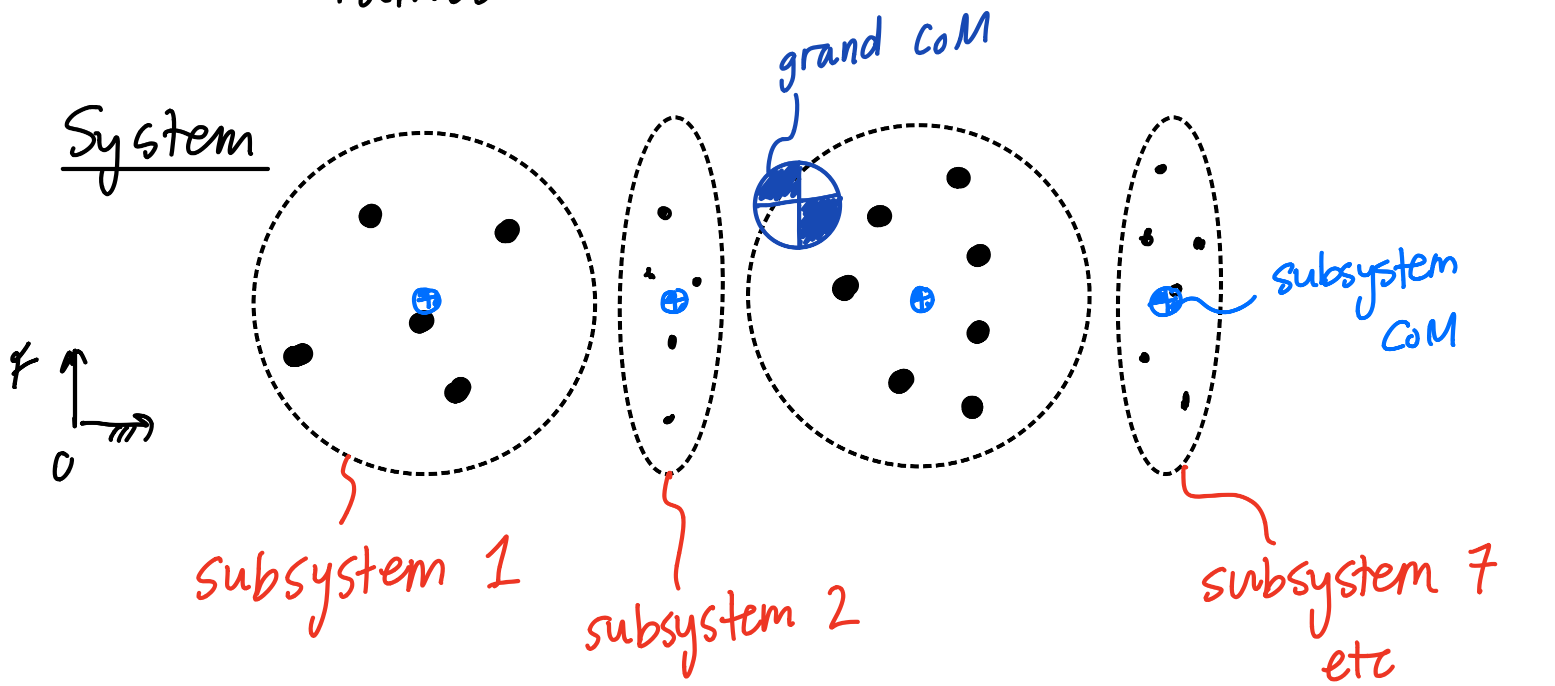
MAE 2030 Lecture 38: 5/12/21

Today: Mechanics Review continued

$\vec{L}$, $\dot{\vec{H}}_{/c}$, E_K, E_P

Frames

System



$$\vec{L} = \sum m_i \vec{v}_i$$

$$= \underbrace{\vec{L}_1 + \vec{L}_2 + \cdots}_{\text{subsystems}}$$

$$= m_{tot} \vec{v}_G$$

LMB: $\sum_{ext} \vec{F} = \dot{\vec{L}}$

$$= m_{tot} a_G$$

$$= \dot{\vec{L}}_1 + \dot{\vec{L}}_2 + \cdots$$

Linear momentum

position of part or rigid object CoM

$$* \quad \sum \vec{M}_{/C} = \sum \vec{r}_{i/C} \times m_i \, \vec{a}_{i/\cancel{C}} + \sum \dot{\vec{w}}_i \, I^{G_i}$$

all particles & rigid bodies

rigid bodies

assumptions + theorem

the only eqn you need for all of mechanics (2D)

$$= \dot{\vec{H}}_{/C}$$

fixed point moving at constant vel

CoM of system

$$\boxed{\vec{H}_{/C} = \sum \vec{r}_{i/C} \times m_i \vec{v}_{i/C} + \sum I^{G_i} \vec{w}_i}$$

$$E_k = \sum \frac{1}{2} m_i |\vec{v}_i|^2 + \sum \frac{1}{2} I^{G_i} |\vec{w}_i|^2$$

particles & rigid objects

$$= \underbrace{\frac{1}{2} m_{tot} |\vec{v}_G|^2}_{E_{KG}} + \underbrace{\sum \frac{1}{2} m_i |\vec{v}_{i/G}|^2}_{E_{K/G}}$$

all mass (includes rigid objects)

Special case: single rigid object

$$E_k = \frac{1}{2} m_{tot} |\vec{v}_G|^2 + \frac{1}{2} I^G |\vec{\omega}|^2$$

- $$\underbrace{\sum_i \vec{F}_i \cdot \vec{v}_i}_{\text{Power}} = \frac{d}{dt}(E_k)$$

all forces (internal & ext)

<u>Special cases:</u>

ex) single particle

$$\vec{F} \cdot \vec{v} = \frac{d}{dt} E_k$$

ex) arbitrary system

$$\left\{ \vec{F}_{tot\ ext} = m_{tot} \cdot a_G \right\}$$

$$\underbrace{\left\{ \sum_{external} F \right\} \cdot \vec{v}_G}_{\text{CoM Power}} = \frac{d}{dt} \underbrace{E_{kG}}_{\frac{1}{2} m v_G^2}$$

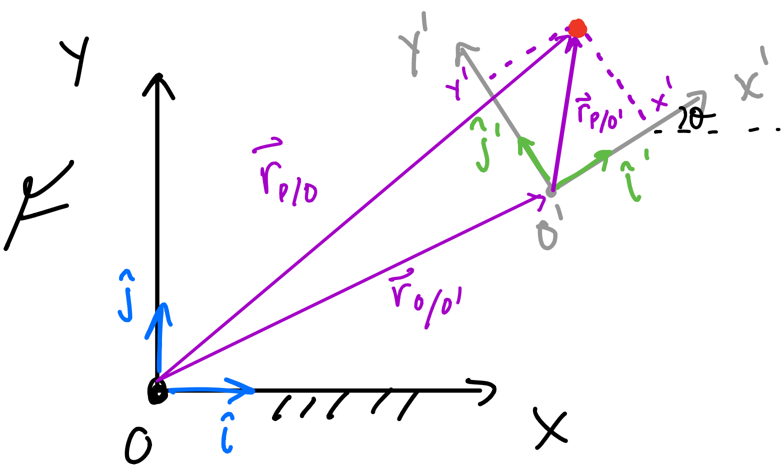
Some forces are <u>conservative</u>

For conservative forces

$$P = \underbrace{-\dot{E}_p}_{\text{rate of decrease of potential energy}}$$

$$W = \int P\,dt = \int \vec{F}\cdot d\vec{r} = \underbrace{-\Delta E_p}_{\text{decrease in potential energy}}$$

Reference Frames:



$$\boxed{\begin{aligned} \dot{\hat{i}}' &= \vec{\omega}_{\beta/\mathcal{F}} \times \hat{i}' \\ \dot{\hat{j}}' &= \vec{\omega}_{\beta/\mathcal{F}} \times \hat{j}' \end{aligned}}$$

$$\vec{v}_{P/\beta} = \dot{x}'\hat{i}' + \dot{y}'\hat{j}'$$

$$\vec{a}_{P/\beta} = \ddot{x}'\hat{i}' + \ddot{y}'\hat{j}'$$

$$\vec{r}_{P/O} = \vec{r}_{O'/O} + \vec{r}_{P/O'}$$

$$\vec{v}_{P/\mathcal{F}} = \underbrace{\vec{v}_{O'/O}}_{\vec{v}_{O'/\mathcal{F}}} + v_{P/\beta} + \vec{\omega}_{\beta/\mathcal{F}} \times \vec{r}_{P/O'}$$

$$\vec{a}_{P/\mathcal{F}} = \vec{a}_{O'/O} + \vec{a}_{P/\beta} + -\overset{\dot{\theta}^2}{\omega^2_{\beta/\mathcal{F}}}\,\vec{r}_{P/O'} + \dot{\vec{\omega}}_{\beta/\mathcal{F}} \times \vec{r}_{P/O'} + 2\vec{\omega}_{\beta/\mathcal{F}} \times \vec{v}_{P/\beta}$$

"Q dot formula"

"transport theorem"

$$\underbrace{\frac{d^{\mathcal{F}}}{dt}\vec{Q}}_{\dot{Q}_x\hat{i}+\dot{Q}_y\hat{j}} = \underbrace{\frac{d^{\mathcal{B}}}{dt}\vec{Q}}_{\dot{Q}_{x'}\hat{i}'+\dot{Q}_{y'}\hat{j}'} + \vec{w}\times\vec{Q}$$