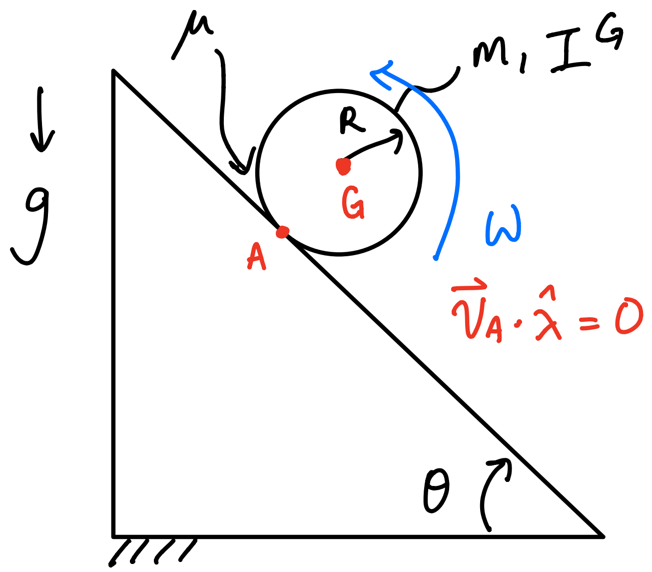
MAE 2030 Lecture #39: last lecture! 5/14/21

Today: Rolling & sliding

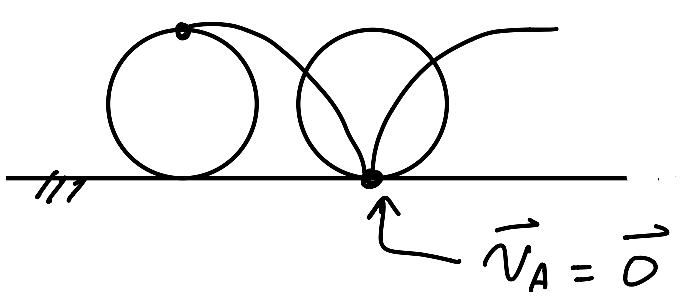
Prelim 3 problem 1 (aka #7)



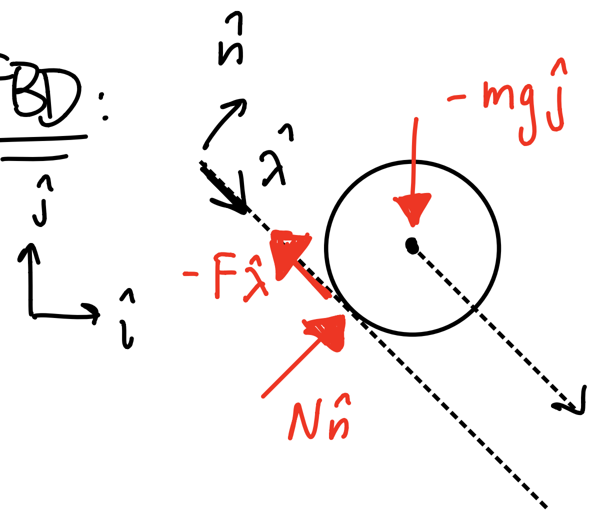
- released from rest

a) assume θ, μ etc such that it slides at first (For how long?)

b) critical case?



FBD:



$|F| \leq \mu N$ when rolling

$|F| = \mu N$ when sliding

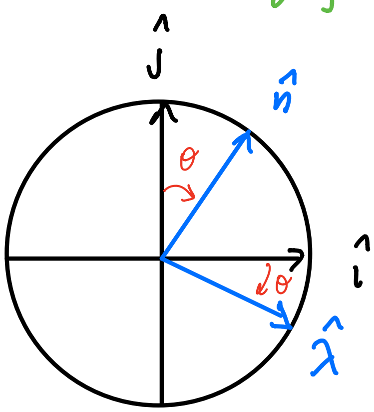
a) assume sliding

LMB:
$$\sum_{ext} \vec{F} = m\vec{a}_G$$

$$\left\{ -mg\hat{j} - \mu \overset{?}{N} \hat{\lambda} + \overset{?}{N} \hat{n} = m \overset{?}{a_G} \hat{\lambda} \right\}$$

2 eqns & 2 unknowns

$$\{\ \}\cdot \hat{n} \Rightarrow -mg\underbrace{\hat{j}\cdot\hat{n}}_{\cos\theta} + N = 0$$



$$\boxed{N = mg\cos\theta}$$

$$\{\ \}\cdot \hat{\lambda} \Rightarrow -mg\underbrace{\hat{j}\cdot\hat{\lambda}}_{-\sin\theta} - \mu N = m a_G$$

$$\cancel{m}g\sin\theta - \mu\cancel{m}g\cos\theta = \cancel{m}a_G$$

$$\boxed{a_G = g(\sin\theta - \mu\cos\theta)}$$ (1)

AMB/G:

$$\sum \vec{M}_{/G} = I^G \underbrace{\dot{\omega}\hat{k}}_{\dot{\vec{\omega}}}$$

$$\left\{ -R\mu \overset{mg\cos\theta}{N}\hat{k} = I^G \dot{\omega}\hat{k} \right\}$$

$$\{\ \}\cdot\hat{k} \Rightarrow \boxed{\dot{\omega} = -\underbrace{\frac{R\mu mg\cos\theta}{I^G}}_{-\alpha}} \quad (7)$$

Solve ODEs of sliding

ODEs
$$\begin{cases} \dot{\omega} = -\alpha \\ \dot{v}_G = a_G \end{cases}$$

constant

ICs: $\omega(0) = 0$, $v_G(0) = 0$

$$\Rightarrow \quad \omega = -\alpha t \quad ✓$$
$$v_G = a_G t \quad ✓$$

Rolling:

$$\vec{v}_A \cdot \hat{\lambda} \equiv v_{slip} = v_s$$

$$\vec{v}_s = (\underset{v_G\hat{\lambda}}{\vec{v}_G} + \underset{\vec{\omega}\times\vec{r}_{A/G}}{\vec{v}_{A/G}})\cdot\hat{\lambda}$$

$$\boxed{v_s = v_G + \omega R = 0}$$

* assuming rolling

Sliding:

$$v_s^{sliding} = a_c t - \alpha R t = 0$$

$\rightarrow$ solve for t for transition time

$$\Rightarrow (a_c - \alpha R)\, t = 0$$

2 cases

$a_c - \alpha R = 0$

implies t can be anything

$a_c - \alpha R \neq 0$

implies $t = 0$

(IC)

Never rolls

critical case:

$$a_c = \alpha R$$

Sub in from 7, 1