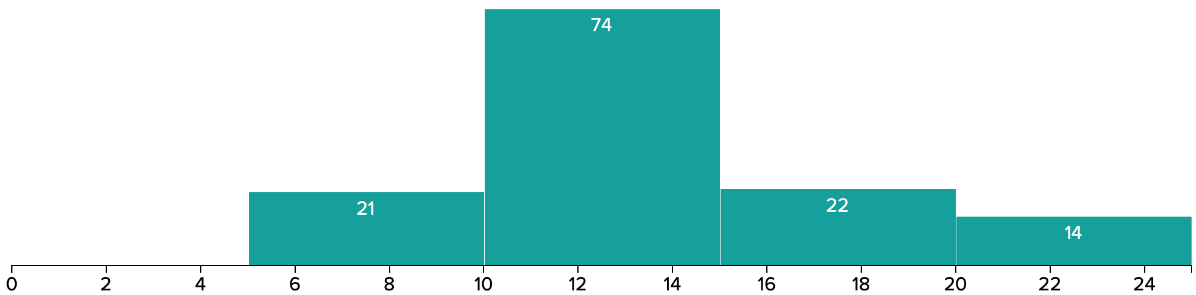
PRELIM 3, PROBLEM 7 (A.K.A. #1)

COMMENTS BY Katherine Cao

Grade Distribution:



MINIMUM	MEDIAN	MAXIMUM	MEAN	STD DEV
6.0	12.0	27.0	13.18	4.52

Common Errors:

- not having overall problem logic
- whether or not the equations were right, some people didn't have the following logic
    1. solving $\overrightarrow{a_G}$ from LMB
    2. solving α from AMB
    3. solving odes (or integrating) to find $\overrightarrow{V_G(t)}$ and $\overrightarrow{\omega(t)}$
    4. stating rolling condition
    5. find t to meet rolling condition

FBD:

Major mistakes include:

- not including gravity
- normal force drawn with incorrect line of action

Minor mistakes include (no deduction):

- free body diagram not free
- normal force drawn inside rather than outside the body
- confusing sign convention for friction force

Calculation errors

The errors in carrying out calculation mainly came from:

- incorrectly assuming sliding means no rotation

- not plugging in friction during sliding, $f = \mu N$
- confusing sign conventions leading to sign errors
- calculation errors such as missing terms while carrying out the calculation
- trig errors, e.g. dividing by cos theta instead of multiplying for normal force

Other errors

- having mostly correct LMB equation(s), but not solving for $\overrightarrow{a_G}$
- assuming friction $f = \mu N$ always holds during rolling
- trying to solve for I_G which was given
- trying to use I about the contact point (contact point is not a fixed point in space)
- thinking $I_P \alpha \hat{k} = \vec{r} \times m\overrightarrow{a_G} + I_G \alpha \hat{k}$ (I_P is I about contact point, shouldn't be used)
- trying to use energy conservation to solve for part (a), while energy was actually not conserved due to frictional sliding
- not checking units or dimensions, which could have helped point out many of the errors above (so could have been corrected during the exam)

Commentary on Problem 8, a.ka. #2, Prelim 3, MAE 2030, Spring 2021

Arnaldo Rodriguez-Gonzalez (with edits by Andy Ruina)

May 16, 2021

1 On Solving the Problem

The problem needs some math competence to take to the end. The general process to solve it, which is illustrated in Prof. Ruina's solution, or my own solution, is to:

1. Draw free-body diagrams of the disk and of the bead.
2. Generate enough equations of motion (2) in order to be able to solve for the power exerted by the motor (this takes a few steps).
3. Set up the work integral $W = \int P\,dt$ and evaluate (best done using a change of variables).

However, even if the "set-up" is correct, it takes some insight to do the work integral. If you set up and solved the ODE for $r(t)$ you could, hypothetically, use that to set up the work integral. But, as far as I could tell, evaluating the integral *and* then evaluating in terms of radius, is basically to hard for most of us mortals. Hence you got 24/25 if you set correctly set up an integral which would, if you were so skilled, give the desired result.

A way to generate a workable integral is by working with the ODEs and not solving them. Then you see which terms show up in the work integral. Finally you use that $\dot{r}dt = \frac{dr}{dt}dt = dr$ and exploit that to perform the integration using a change of variables (You can see the process in the solutions.). Only one of you did this.

However, most of you didn't set up the integral, so the difficulty in evaluating it was never an issue. Most people didn't even generate an FBD of the disk — presumably, the fact that variables weren't declared for the mass/inertia of the disk made people think that forces/moments on the disk were irrelevant. However, angular momentum balance for the disk was key to solving the problem; it's just that the mass and inertia of the disk drop out of the equations because of the constant ω motion.

2 Grading Scheme

I graded based on whether or not you got to a place where you could correctly solve the problem given a computer, etc. I also gave +3 points to anyone who actually (correctly) tried to go through the process of solving the ODE for $r(t)$. The net distribution of points was the sum of three components:

1. Did you generate both free body diagrams correctly/did you account for forces correctly? ($\sim +8$)
2. Were you able to find 2 correct equations of motion for the bead/disk/etc.? ($\sim +8$)
3. Did you write a correct expression for the work in terms of the system parameters? ($\sim +8$)

2.1 Free-Body Diagrams

Most of you generated one free-body diagram correctly, almost always for the bead, which netted you +6 points. Some of you, however, generated free-body diagrams where gravity wasn't neglected, or the normal/friction forces were ignored or pointed in the wrong direction. Unfortunately, this generated errors that propagated down through to the equations of motion—those who at least generated an FBD of some kind got +3. Not much more to say here.

2.2 Equations of Motion

Most of you either generated only one equation of motion, which isn't enough for you to calculate the power, or generated two (with mistakes somewhere in there). Most of you did your LMB/AMB analyses on the bead, which could net you the answer, but you'd need to go through the constants/inverting nightmare — I gave those who did this process correctly full credit on this section. One mistake *lots* of people made: you wrote down the acceleration of the bead as if it was circular motion ($\vec{a} = r\ddot{\theta}\hat{\theta} - r\dot{\theta}^2\hat{r}$), but it's not circular motion. The bead spirals out, so the other terms matter! This misled a few people into thinking they only needed one EoM; a deadly mistake, since you have only one equation of motion with mistakes in it, netting those a (+3). People also mixed up directions of acceleration components, or forgot a factor of two in Coriolis terms. In short, you got +4 for each correct equation of motion (for a total of +8), with a point or two taken off if you had an error in one/two of them. Review that polar coordinate acceleration formula!

2.3 Work

Okay everyone, we need to really go over what work is — nearly no one was able to correctly define what work was for this system, even without considering the state variables! The single biggest mistake everyone made is thinking the path distance of the bead is $R_2 - R_1$; this is wrong! This would mean the bead travels in a straight line without spinning around, which is naturally untrue. The real expression people needed to use to find work is as the integral of power of all forces and moments on the disk,

$$W = \int_{t_1}^{t_2} P dt, \quad P = \sum \vec{F}_i \cdot \vec{v}_i + \sum \vec{M}_i \cdot \vec{\tau}_i$$

Lots of people attempted to define work (a scalar) by multiplying a vector with a scalar, which is vecor so can't possibly be work. Others generated expressions for the work that did not have units of energy. Some correctly tried to find work as the change in kinetic energy of the whole system, but then failed to consider the full kinetic energy of the bead and that there is negative internal frictional work in the system. Often people ignored the change in the angular velocity of the bead, or ignored the radial velocity of the bead as a result of the circular motion fallacy I mentioned in the previous section . Only a handful of you got a correct expression. I decided to be generous and give half credit (+4) to anyone who gave an expression for the work that is feasibly correct in *some* system but incorrect in this one for whatever reason. If you got an expression for work that made sense, applied to this system in a well-developed way but had mistakes, I gave you a +6. If there is anything you take away from this commentary, it's that **you should practice work calculations before the final**.

3 Statistics

I did the statistics on the grades myself; Gradescope tends to bias the bins to make graders look better and the grade distributions be as Gaussian as possible. The mean grade is roughly 15/25, with a "standard deviation" of about 5.83. You can see it below in Figure 1.

Using my (hopefully less biased) binning, the distribution of grades is not well-described by a Gaussian —if I were to describe it, it seems like a roughly uniform distribution with a few gaps due to the granularity of the way I assigned points, and no grades below 4 since most of you at least attempted to generate a free-body diagram or equations of motion, or defined work in an elementary way.

4 Afterword

You've got this — you're almost to the finish line! Use the comments I and the fellow TA's provide to help you do well on the final. I believe in you!

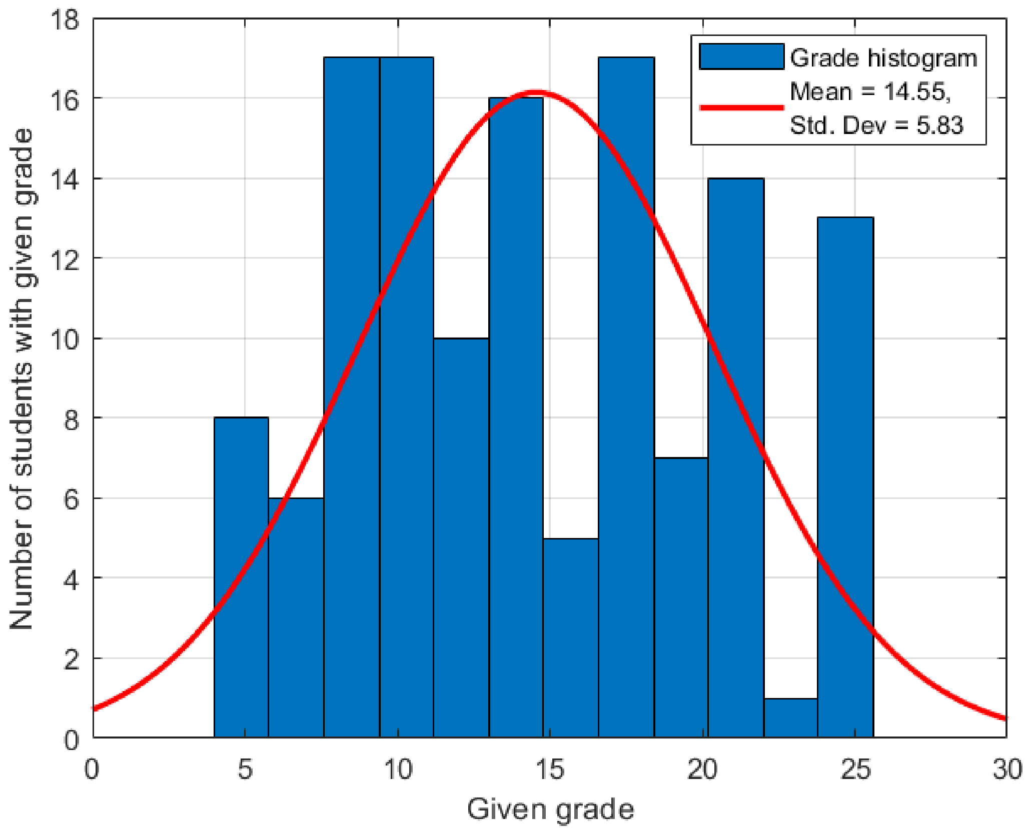


Figure 1: Grade distribution for Prelim 3 Problem 8 in blue, with best-fit Gaussian in red. The number of bins is equal to the rounded square root of the number of grades (134ish).

Prelim 3 PROBLEM 9 REFLECTION
MAE 2030 Spring 2021
Maggie Liu

General:
Part A of this problem seemed confusing to most students. Very few students got it almost or completely correct. The errors in this problem were mostly related to writing the AMB and figuring out how to derive the acceleration at point H. Several students overcomplicated the problem by applying the AMB at point B, which led to mistakes in their equation with either the sum of moments about that point or incorrect terms on the right-hand side. Part B had better results for correctness, in comparison to part A. There were a few students who applied the parallel axis theorem and go the right answer. However, there were also a few that attempted to write down a moment of inertia just from memory without showing any work.

Part A: Finding acceleration at point H after cutting CD
Part A was worth 17 points, as it required more applications of what was covered in lectures. Grading for part A was broken down into two different methods. If your solution looked mostly right, points were deducted for missing or incorrect components. This includes missing FBD or algebraic errors. Method 2 was for mostly wrong solutions, in which points were added for having correct parts.

No mechanical analysis can be done without an FBD. Since this problem is asking for acceleration after the cut, it was important that there was an FBD after the cut to do the calculations. 5 points given for having the FBD for after the cut with correct forces and coordinates. For solutions generated with just an FBD before the cut, 2 points were given. An FBD before the cut will not give the complete and correct analysis needed for this problem, however, it was important in calculating the tension force before the cut using an LMB. For having found the correct tension force, either using symmetry and inspection or the LMB before the cut, 1 point was given.

Using the FBD after the cut, students were intended to use the LMB to solve for $\overline{a_G}$, analyzing that tension force in AB is the same as it was before the cut. 3 points were given for correctly calculating this value.

The most common mistake was setting up the incorrect AMB. Many students had wanted to take the AMB about point B (shown below). However, this led to many errors in determining what terms are on the right-hand side of the equation and in the end → the wrong ωdot. The most straightforward, and I presume as the easiest way to solve this problem, would be using the AMB about point G (as shown in the prelim solutions), therefore the equation to solve for $\overline{a_H}$ would be $\overline{a_H} = \overline{a_G} + \overline{a_{H/G}}$. This makes use of the previously calculated $\overline{a_G}$ from the LMB after the cut. For having written the correct equation down for solving for $\overline{a_H}$, whether this be using acceleration at point B or point G as a reference, 1 point was given. The acceleration of H in reference to another point required you to find ωdot. $\omega dot$is found using the AMB, therefore 3 points were given for having written the correct equation down and having found the correct ωdot.

$$\sum \vec{M}_{/B} = \dot{\vec{H}}_{/B}$$
$$\vec{r}_{G/B} \times \vec{F}_{G} = \vec{r}_{G/B} \times m\vec{a}_{G} + I^{G}\dot{\omega}\hat{k}$$

Some students had also found the angular acceleration and assumed that that was the answer to the problem. The question is asking for the acceleration of point H, not the angular acceleration.

A rather surprising but common mistake was finding the cross product. This could have been a mistake in finding the cross product itself or leaving behind a sign.

$$\hat{k} \times \hat{i} = \hat{j} \quad \Rightarrow -\hat{k} \times \hat{i} = -\hat{j}$$
$$\hat{k} \times \hat{j} = -\hat{i} \quad \Rightarrow -k \times -\hat{j} = -\hat{i}$$

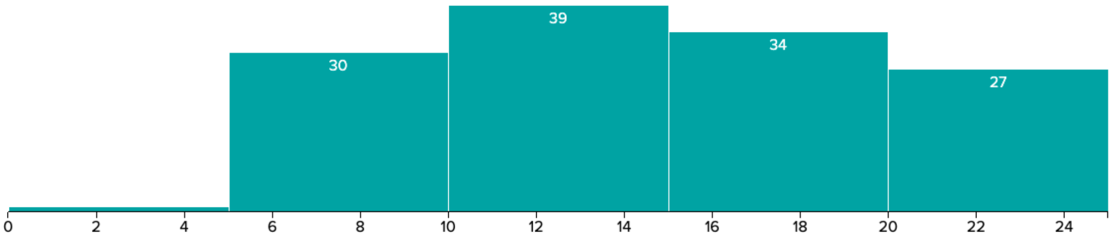
Part B: Moment of Inertia about G

The correct method to solving this problem is using the integral equation for moment of inertia about the center of gravity. There were many errors with either having the wrong bounds, dm conversion to dxdy, and r. For incorrect bounds, 3 points were deducted as this was key factor in understanding the problem and what moment of inertia about G meant. For incorrect dm and r, 2 points were deducted for each. The most common mistake was using 0 to L as the bounds of integration. However, since this question is asking for moment of inertia about the center of gravity, the bounds should be from -L/2 to L/2.

For students who used another method, such as the parallel axis theorem, 5 points are given. There were a few that simply wrote down an equation based on memory. Incorrect “guesses” without any work were given 0 points.

Statistics

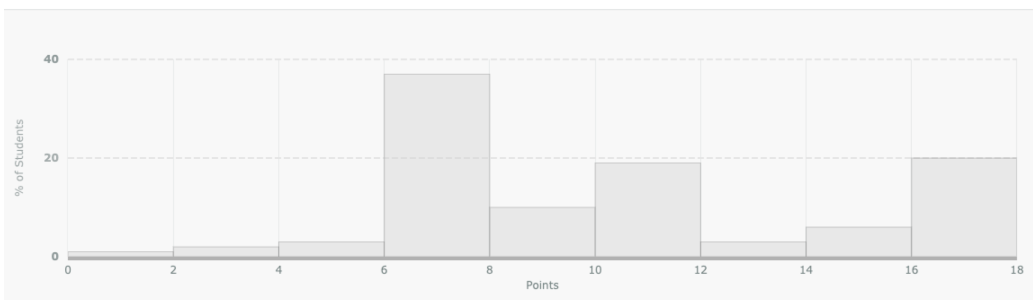
Overall:



MINIMUM	MEDIAN	MAXIMUM	MEAN	STD DEV
4.0	14.0	25.0	14.6	5.7

Part A:

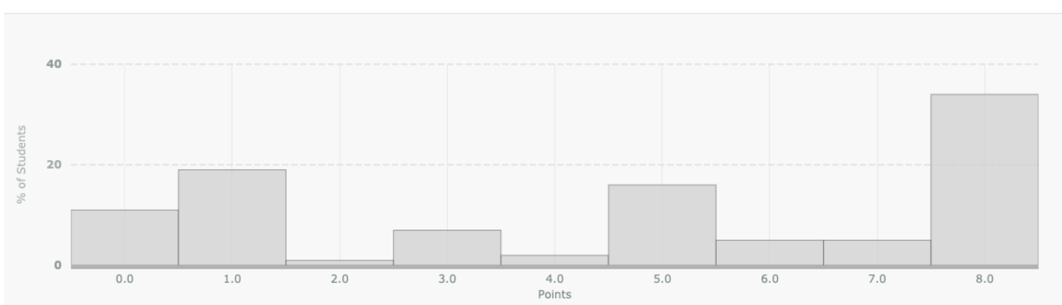
< Q 1.1: (a) Acceleration of point H 17 points Grade Question



MINIMUM	MEDIAN	MAXIMUM	MEAN	STD DEV
1.0	9.0	17.0	9.95	4.15

Part B:

< Q 1.2: (b) moment of Inertia about G 8 points Grade Question



MINIMUM	MEDIAN	MAXIMUM	MEAN	STD DEV
0.0	5.0	8.0	4.72	3.07